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On the generalised Fant equation

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ABSTRACT

An analysis is made of the fluid structure interactions involved in the production of voiced speech. It is usual to avoid time consuming numerical simulations of the aeroacoustics of the vocal tract and glottis by the introduction of Fant's 'reduced complexity' equation for the glottis volume velocity Q [G. Fant, *Acoustic Theory of Speech Production*, Mouton, The Hague 1960]. A systematic derivation is given of Fant's equation based on the nominally exact equations of aerodynamic sound. This can be done with a degree of approximation that depends only on the accuracy with which the time varying flow geometry and surface acoustic boundary conditions can be specified, and replaces Fant's original 'lumped element' heuristic approach. The method determines all of the effective 'source terms' governing Q. It is illustrated by consideration of a simplified model of the vocal system involving a self sustaining single mass model of the vocal folds, that uses free streamline theory to account for surface friction and flow separation within the glottis. Identification is made of a new source term associated with the unsteady vocal fold drag produced by their oscillatory motion transverse to the mean flow.

1. Introduction

Voiced speech involves quasi periodic vibrations of the vocal folds at frequencies $f_0 \sim 100-300$ Hz excited by a slowly modulated overpressure produced by contraction of the lung cavity by the respiratory muscles. The narrow airway between the vocal folds is called the glottis, and fold vibration causes the glottis cross sectional area to oscillate thereby pulsing the airflow through the glottis with volume velocity $Q(t)$, say, at characteristic frequency f_0 (where t denotes time) [1 3]. The glottis accordingly behaves as a monopole sound source of strength Q that radiates into the supraglottal section of the vocal tract and thence out into 'free space' via the nasal and mouth cavities.

Fant [1] recognised that the principal action of the vocal folds is to control the volume velocity Q of the air stream, and that it is the unsteady stream flow that should be identified as the source of the sound *not* the surface vibration of the folds. The flow emerges from the glottis in the form of a jet, and Fant's original analysis was based on the simplified model of the vocal system illustrated in Fig. 1a, where the supraglottal tract is represented by a uniform duct that radiates sound from its open end to a listener in free space. The subglottal region also is modelled by a uniform duct terminating in the lung complex. Motion of the vocal folds is maintained by muscular tension and by alternating forces consisting essentially of the subglottal over pressure when the glottis is closed, and in the open state by a negative Bernoulli suction force produced by flow through the glottis.

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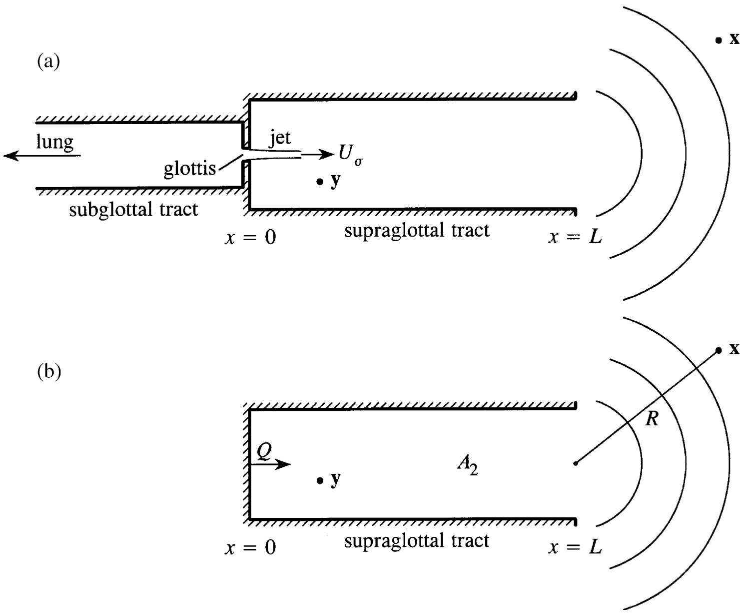

Fig. 1. (a) Idealised representation of the subglottal and supraglottal sections of the vocal tract, each modelled as uniform rigid ducts. (b) The corresponding source-filter representation of sound produced in free space at **x** by the glottis volume velocity $Q(t)$.

The effective volume source strength $Q(t)$ determines the voiced acoustic field. It must therefore incorporate into its governing equation all mechanisms that contribute to sound generation. In Fant's original model [1] the equation for Q was derived by a simple lumped parameter argument for excitation by a constant subglottal overpressure p_{sg} in the form

$$\overline{\ell}(t)\frac{dQ}{dt}+R(t)Q=p_{sg}, \tag{1}$$

where $\overline{\ell}=\rho_0 L/A_g$ is the glottis inductance, and the resistive term R is given by

$$R=\rho_0 Q/2A_g^2+R'. \tag{2}$$

In these equations ρ_0 is the mean air density, L, A_g are respectively the streamwise length and cross sectional area of the glottis, and R' represents the effects of viscous losses.

The lung glottis monopole representation of voiced speech was also formulated and extended by Flanagan [2] and Ishizaka and Flanagan [4]. In particular the first fully interactive theory of voicing, which coupled aerodynamically self sustaining oscillations of the vocal folds with sound production described by a modified form of the Fant equation (1) was developed in [4]. This theory made use of a 'two mass' physical model of the vocal folds which introduced a suitable phase lag that permitted the transfer of energy from the mean flow to maintain the fold oscillations.

Many current and recent studies of voicing continue to make use of some form of the Fant equation coupled to an equation or equations of motion of a structural model of the vocal folds. Thus, Fulcher et al. [5] proposed an equation for fold motion containing a heuristic negative Coulomb damping term to represent aerodynamic excitation. Zanartu et al. [6] achieved self sustaining oscillations by invoking negative damping within the glottis caused by a prescribed unsteady discharge coefficient for the glottal jet and by acoustic feedback from the vocal tract. The importance of acoustic feedback is still uncertain. Titze [7] has asserted that it must normally be relatively weak. At higher frequencies, approaching the first formant, however, it has been argued that the voicing monopole can interact with vocal tract resonances to produce undesirable, involuntary and abrupt changes in frequency [8 11]. But many estimates of the influence of acoustic feedback are frequently based on *ad hoc* approximations of the coupling of sound and surface tissue in the glottis (e.g. [6,11 13]).

Recent numerical simulations [14 20] of the flow and glottal motions aim to make no assumptions about the nature of feedback coupling, which should emerge from a more or less precise numerical treatment of the equations of motion. They make use of the full equations of motion and can in principle supply valuable insight into the fluid structure acoustic interactions. However, 'one off' numerical treatments of this kind tend to be computationally intensive and often cannot be run in a timely manner for more than one or two voicing cycles, and for the purposes of routine analysis the very global and detailed nature of the computations actually emphasise the need for a 'reduced complexity' model of the kind provided by the Fant equation. This objective can be formally achieved by using the same exact equations, but rearranged in the manner of Lighthill's *acoustic analogy* [21 25] with the object of supplying an analytical representation of the voiced sound in terms of the surface and flow sources.

In a reduced complexity analysis it is assumed that the relevant acoustic frequencies are sufficiently low that the sound propagates one dimensionally (as 'plane' waves) in the vocal tract. The equations of motion and an acoustic Green's function G can then be used to express the overall acoustic volume velocity Q at an arbitrary position in the supraglottal tract as the convolution $Q = G{*}\mathcal{S}$ of G and the acoustic sources $\mathcal{S}$. When the equations of motion are further used to recast the sources $\mathcal{S}$ as a combination of components $\mathcal{S}_g(Q)$ dependent on Q near the glottis, and a component $\mathcal{S}_l(p_l)$ determined by properties defining the state of lung contraction, where p_l is the overpressure created by the lungs, we then obtain

$$Q = G{*}\mathcal{S}_g(Q) + G{*}\mathcal{S}_l(p_l). \tag{3}$$

The sources $\mathcal{S}_g$ are nonlinear and non zero only in the neighbourhood of the glottis; $\mathcal{S}_l$ is non zero in the lungs. Eq. (3) is an integral equation involving integrations over both space and time. The limiting form of the equation as the field position approaches the glottis transforms it into the required reduced complexity equation for Q at the glottis, which we identify as the *generalised Fant equation* because it governs the glottal volume flow just as the original Fant equation (1) does.

The exact equations of fluid motion are used in each stage of this derivation; it is only after the general form of the equation has been obtained that approximations are introduced to evaluate or estimate particular components of the equation in terms of the flow and the vocal tract geometry and mechanical properties. This is the procedure advocated and implemented to great acclaim by Lighthill [21] in his theory of aerodynamic sound, who emphasised that, because sound is generally a very small by product of a flow, approximations should be delayed until the very last stages of an acoustic calculation to avoid discarding apparently small, but nevertheless important source terms. It is not the procedure adopted hitherto in the voicing literature, where approximations tend to be introduced at an early stage of problem formulation, nor in the numerical/analytical modelling of wind instruments and engineering devices subject to fluid structure excitation of resonances, although such approximate methods with appropriate empirical input are often very effective in predicting the resonant response [26 29]. The method was discussed by Howe and McGowan [25] in a case where the motion of the glottis was prescribed independently of the acoustic problem.

In this paper the general procedure is described in detail and illustrated by application to a simple flow coupled single mass model of the vocal folds discussed in [30], which is similar to that investigated experimentally by Barney et al. [31]. The flow structure interaction involves an intermittent suction force that acts to pull the folds together, and is associated with a fluctuating component of drag produced by variations in the separation point of the jet formed in the glottis (cf. [32]).

The general representation of the voiced sound is discussed in Section 2 using the full aeroacoustic equation applicable in the presence of deformable boundaries. Application is first made to classical source filter theory [1] in Section 3. A further particular example is then discussed (Sections 4 and 5) that uses the single mass vocal folds model of [30]; the new surface source of sound is discussed and incorporated into the generalised Fant equation for this model.

2. General representation voiced sound

2.1. The aerodynamic sound equation

Sound in air is produced by moving boundaries, by vorticity within a flow, and by the interaction of vorticity with the boundaries. In a first approximation the fluid motion involved in the production of speech can be taken to be *homentropic*. This does not mean that dissipative processes within the fluid are negligible, but that their effects are confined to regions of strong shear at the boundaries, whereas in the body of the fluid sound propagates as if the fluid were inviscid. Then the momentum equation for the motion at $\mathbf{x} = (x,y,z)$ within the fluid at time t assumes Crocco's form [24,33]

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla B = \quad \boldsymbol{\omega}\wedge\mathbf{v} \quad \frac{\eta}{\rho}\,\mathrm{curl}\,\boldsymbol{\omega}, \tag{4}$$

where $\mathbf{v}$ is the velocity, $\boldsymbol{\omega} = \mathrm{curl}\,\mathbf{v}$ is the vorticity, η is the shear coefficient of viscosity, and B is the total enthalpy, given by

$$B = \int \frac{\mathrm{d}p}{\rho} + \frac{1}{2}v^2 \tag{5}$$

in homentropic flow, where p is the pressure and $\rho \equiv \rho(p)$ is the fluid density.

The corresponding equation governing sound production (obtained from (4) and the equation of continuity) is [23,24]

$$\left(\rho\frac{D}{Dt}\left(\frac{1}{c^2}\frac{D}{Dt}\right) \quad \frac{\partial}{\partial x_j}\left(\rho\frac{\partial}{\partial x_j}\right)\right)B = \mathrm{div}(\rho\boldsymbol{\omega}\wedge\mathbf{v}), \tag{6}$$

where $\boldsymbol{\omega}\wedge\mathbf{v}$ is the *Lamb* vector and c is the local speed of sound, and the repeated subscript j implies the usual summation over all three spatial coordinates.

In the absence of vorticity and moving boundaries the Bernoulli equation implies that B is equal to a constant throughout the fluid that may be assumed to vanish. Outside the source region the unsteady motion is entirely irrotational, and is represented by a velocity potential $\varphi(\mathbf{x},t)$, in terms of which $B \equiv \quad \partial\varphi/\partial t$ determines the amplitude of the propagating sound waves. B is related to the acoustic pressure in the far field by

$$\frac{1}{\rho}\frac{\partial p}{\partial t} = \frac{DB}{Dt}. \tag{7}$$

When the mean flow Mach number ($M \sim v/c$) is small in the acoustic region, the sound pressure $p=\rho_0 B$, where ρ_0 is the mean density.

When $M^2 \ll 1$ the vortex sound equation (6) takes the simplified form

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x_j^2}\right)B=\operatorname{div}(\boldsymbol{\omega}\wedge\mathbf{v}), \tag{8}$$

where c_0 is the uniform mean sound speed.

2.2. Formal representation of the sound

The causal solution $B(\mathbf{x},t)$ of Eq. (6) can be expressed in terms of Green's function $G(\mathbf{x},\mathbf{y},t,\tau)$, which is an 'advanced potential' that satisfies

$$\left(\rho\frac{D}{D\tau}\left(\frac{1}{c^2}\frac{D}{D\tau}\right)-\frac{\partial}{\partial y_j}\left(\rho\frac{\partial}{\partial y_j}\right)\right)G=\rho\delta(\mathbf{x}-\mathbf{y})\delta(t-\tau),\quad G=0 \text{ for } \tau>t, \tag{9}$$

wherein it will be convenient to adopt the notation $\mathbf{y}=(x',y',z')$. This represents a disturbance that propagates as an 'incoming' wave as a function of $(\mathbf{y},\tau)$ towards the singularity at $\mathbf{y}=\mathbf{x}$ on the right hand side of (9), arriving at $\tau=t$ and vanishing thereafter.

Replace the variables $(\mathbf{x},t)$ in (6) by the 'dummy variables' $(\mathbf{y},\tau)$ and multiply the equation by $G(\mathbf{x},\mathbf{y},t,\tau)$; subtract the result from the product of $B(\mathbf{y},\tau)$ and Eq. (9) and integrate over $-\infty<\tau<\infty$ and over the whole of the spatial region $V(\tau)$, say, occupied by the fluid at time τ. The result can be rearranged into the form [23,24,34]

$$\begin{aligned}\rho B(\mathbf{x},t)=&\int_{-\infty}^{\infty}\frac{\partial}{\partial\tau}\left\{\int_{V(\tau)}\frac{\rho}{c^2}\left[B\frac{DG}{D\tau}-G\frac{DB}{D\tau}\right]\mathrm{d}^3\mathbf{y}\right\}\mathrm{d}\tau+\int_{-\infty}^{\infty}\int_{V(\tau)}\frac{\partial}{\partial y_j}\left(\rho G\frac{\partial B}{\partial y_j}-\rho B\frac{\partial G}{\partial y_j}\right)\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau\\&+\int_{-\infty}^{\infty}\int_{V(\tau)}\frac{\partial}{\partial y_j}(G\,\rho(\boldsymbol{\omega}\wedge\mathbf{v})_j)\,\mathrm{d}^3\mathbf{y}\mathrm{d}\tau-\int_{-\infty}^{\infty}\int_{V(\tau)}\frac{\partial G}{\partial y_j}\,\rho(\boldsymbol{\omega}\wedge\mathbf{v})_j\,\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau.\end{aligned} \tag{10}$$

The first term on the right of this formula is deduced by careful application of the equation of continuity and the condition that the fluid and boundary have the same normal component of velocity at their interface. Its validity can also be confirmed more intuitively by first writing the spatial integral as an elementary sum over fluid particles, each of invariable mass $\rho\delta^3\mathbf{y}$, and each of which is displaced by motion of fluid boundary subject to $D(\rho\delta^3\mathbf{y})/D\tau=0$.

Now G and $DG/D\tau$ both vanish at $\tau=+\infty$, and causality permits one to assume that B and $DB/D\tau$ vanish at $\tau=-\infty$. Therefore, there is no contribution from the first integral on the right of (10). The second and third integrals are simplified by the use of the divergence theorem, yielding

$$\rho B(\mathbf{x},t)=\int_{-\infty}^{\infty}\oint_{S(\tau)}\rho\left[B\frac{\partial G}{\partial\mathbf{y}}-G\left(\frac{\partial B}{\partial\mathbf{y}}+\boldsymbol{\omega}\wedge\mathbf{v}\right)\right]\cdot\mathrm{d}\mathbf{S}(\mathbf{y})\,\mathrm{d}\tau-\int_{-\infty}^{\infty}\int_{V(\tau)}\rho\frac{\partial G}{\partial\mathbf{y}}\cdot\boldsymbol{\omega}\wedge\mathbf{v}\,\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau, \tag{11}$$

where the surface integrals are over the physical boundary $S(\tau)$ of the fluid at time τ, with the surface element $\mathbf{dS(y)}$ directed into the fluid.

Further simplification is achieved by taking the particular Green's function that satisfies (9) and the additional condition that $\partial G/\partial y_n=0$ on $S(\tau)$, where y_n is a local normal coordinate on $S(\tau)$ directed into the fluid. Then, application of the Crocco equation (4) supplies the representation

$$\rho B(\mathbf{x},t)=\int_{-\infty}^{\infty}\oint_{S(\tau)}\left[\rho G\frac{\partial\mathbf{v}}{\partial\tau}-\eta\frac{\partial G}{\partial\mathbf{y}}\wedge\boldsymbol{\omega}\right]\cdot\mathrm{d}\mathbf{S}(\mathbf{y})\,\mathrm{d}\tau-\int_{-\infty}^{\infty}\int_{V(\tau)}\rho\frac{\partial G}{\partial\mathbf{y}}\cdot\boldsymbol{\omega}\wedge\mathbf{v}\,\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau. \tag{12}$$

The surface integral is the contribution to the radiation from accelerated motion of the boundary and from frictional forces on the boundary; the volume integral is the contribution from vorticity within the fluid.

This result can be simplified for use in the low Mach number vocal tract régime, where variations in the fluid density are very small corresponding to the approximation of Eq. (8), by setting $\rho=\rho_0$, so that

$$B(\mathbf{x},t)=\int_{-\infty}^{\infty}\oint_{S(\tau)}\left[G\frac{\partial\mathbf{v}}{\partial\tau}-\nu\frac{\partial G}{\partial\mathbf{y}}\wedge\boldsymbol{\omega}\right]\cdot\mathrm{d}\mathbf{S}(\mathbf{y})\,\mathrm{d}\tau-\int_{-\infty}^{\infty}\int_{V(\tau)}\frac{\partial G}{\partial\mathbf{y}}\cdot\boldsymbol{\omega}\wedge\mathbf{v}\,\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau, \tag{13}$$

where $\nu=\eta/\rho_0$ is the kinematic viscosity.

3. Source-filter theory

The simplest representation of voicing is in terms of the effective 'monopole' source strength $Q(t)$ of the glottis. A steady overpressure applied in the subglottal region by contraction of the lung cavity produces a flow through the glottis with volume velocity $Q(t)$, whose time dependence is controlled by the mechanical properties of the vocal folds and their interactions with the air flow. The 'source filter' representation of this mechanism is obtained by assuming in (13) that the supraglottal tract is acoustically closed at the glottis, which is then replaced by a point volume source of prescribed strength $Q(t)$.

This procedure is applicable in principle to any geometrical configuration of the vocal tract. For the purposes of illustration, however, we consider the simplest possible model depicted in Fig. 1a. The supraglottal tract is represented by a uniform, hard walled duct of length L and cross sectional area A_2, radiating from its open end to an observer at position $\mathbf{x}$ in free space. The subglottal tract consists of a uniform duct of cross section A_1 that terminates in the lung complex. Air from the lungs enters the supraglottal system as a jet of volume velocity $Q(t)$ through the glottis of time dependent area A_g. Take the x axis to coincide with the axis of symmetry of the glottis, with the coordinate origin on its anterior face, as indicated in the figure.

To calculate voiced speech the source filter theory replaces this fluid acoustic system by the simplified model shown in Fig. 1b, where the role of the glottis is assumed by a volume source of net strength $Q(t)$ on the surface of an otherwise rigid boundary. The source is also responsible for maintaining the jet flow (not shown explicitly in Fig. 1b), in other words it is a source of both fluid volume and of x momentum. The contribution from viscous stresses to the surface integral in (13) is assumed to be contained within the definition of Q and to be negligible in the supraglottal tract, so that

$$\int_{-\infty}^{\infty}\oint_{S(\tau)}\left[G\frac{\partial \mathbf{v}}{\partial \tau}\quad \nu\frac{\partial G}{\partial \mathbf{y}}\wedge\boldsymbol{\omega}\right]\cdot \mathrm{d}\mathbf{S}(\mathbf{y})\,\mathrm{d}\tau \approx \int_{-\infty}^{\infty} G(\mathbf{x},\mathbf{0},t,\tau)\frac{\partial Q}{\partial \tau}(\tau)\,\mathrm{d}\tau. \tag{14}$$

The remaining volume integration in (13) involving the Lamb vector $\boldsymbol{\omega}\wedge\mathbf{v}$ supplies a component of the sound that is always $\sim O(M^2)$ smaller than (14). This is because its contribution in the presence of a rigid boundary is equivalent to that generated by a quadrupole formed by the jet and its image in the boundary when the latter is formally removed [21,22,24].

To confirm these conclusions explicitly we use the following expression for Green's function:

$$G(\mathbf{x},\mathbf{y},t,\tau)\approx\frac{1}{8\pi^2 R}\int_{-\infty}^{\infty}\frac{\cos(k_0x')\mathrm{e}^{-\mathrm{i}\omega(t-\tau-R/c_0)}\,\mathrm{d}\omega}{\cos\{k_0(L+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}},\quad \mathbf{y}=(x',y',z'),\ k_0=\omega/c_0, \tag{15}$$

where R is the distance of the observer position at $\mathbf{x}$ in free space from the centroid of the open 'mouth' of the supraglottal tract at x=L, and ℓ'_E ($\sim 0.8\sqrt{A_2/\pi}$) is the end correction of the mouth [24,35]. The representation (15) is formally valid when R is much larger than the diameter of the mouth, and for the usual case where acoustic wavelengths $\sim 2\pi/k_0 \gg \sqrt{A_2}$, so that only plane waves can propagate within the duct.

The acoustic pressure $p(\mathbf{x},t)\approx\rho_0 B(\mathbf{x},t)$, and the component associated with the glottis source Q is given from (14), (15) by

$$p(\mathbf{x},t)\approx\frac{\rho_0}{4\pi R}\int_{-\infty}^{\infty}\frac{\mathrm{i}\omega\hat{Q}(\omega)\mathrm{e}^{-\mathrm{i}\omega(t-R/c_0)}\,\mathrm{d}\omega}{\cos\{k_0(L+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}}, \tag{16}$$

where $\hat{Q}(\omega)=(1/2\pi)\int_{-\infty}^{\infty}Q(\tau)\mathrm{e}^{\mathrm{i}\omega\tau}\,\mathrm{d}\tau$ is the Fourier transform of the volume velocity. By defining the Fourier transform $\hat{p}(\mathbf{x},\omega)$ of the pressure in the same way, we arrive at the familiar frequency domain, source filter representation of the voiced pressure:

$$\hat{p}(\mathbf{x},\omega)\approx\frac{\mathrm{i}\omega\rho_0\hat{Q}(\omega)\mathrm{e}^{\mathrm{i}k_0R}}{4\pi R\cos\{k_0(L+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}}. \tag{17}$$

The cosine in the denominator vanishes at complex values of ω very close to the formants ω_1, ω_2, ω_3, …, but having small negative imaginary parts that account (according to the present idealised model of the supraglottal tract) for damping of sound in the vocal tract by the radiation from the mouth. These formants are governed by the 'running' large scale characteristics of the supraglottal tract. The characteristics of the glottal volume velocity $\hat{Q}(\omega)$ depend also on the properties of the vocal folds and the subglottal system. The combined characteristics determine in (17) the nature of the voiced sounds.

The corresponding expression for the frequency domain pressure $\hat{p}_j(\mathbf{x},\omega)$, say, produced by the jet source (the final term on the right of (13)) is similarly shown to have the form

$$\hat{p}_j(\mathbf{x},\omega)\approx\frac{\rho_0k_0^2\hat{\mathcal{L}}(\omega)\mathrm{e}^{\mathrm{i}k_0R}}{4\pi R\cos\{k_0(L+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}}, \tag{18}$$

where

$$\mathcal{L}(\tau)=\int_{V(\tau)}x'(\boldsymbol{\omega}\wedge\mathbf{v})_1(\mathbf{y},\tau)\,\mathrm{d}^3\mathbf{y}. \tag{19}$$

Now, $\nu\sim Q/\ell_j^2$, where ℓ_j is a characteristic diameter of the jet issuing from the glottis, and it therefore follows that the order of magnitude of $\hat{p}_j$ is smaller than (17) by a factor of order $\sim k_0\ell_jM$ ($\sim O(M^2)\ll 1$).

The procedure outlined in Section 1 for the formal derivation of the generalised Fant equation is easily applied in the present approximation. But it is evident that it must lead to the tautology Q=Q. Indeed, when source and observer positions $\mathbf{y}=(x',y',z')$, $\mathbf{x}=(x,y,z)$ both lie within the supraglottal tract of Fig. 1b, it is readily shown for compact cross sections that

$$G(\mathbf{x},\mathbf{y},t,\tau)\approx\frac{1}{2\pi A_2}\int_{-\infty}^{\infty}\frac{\cos(k_0x')\sin\{k_0(L\quad x+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}\mathrm{e}^{-\mathrm{i}\omega(t-\tau)}\,\mathrm{d}\omega}{k_0\cos\{k_0(L+\ell'_E+\mathrm{i}k_0A_2/4\pi)\}}, \tag{20}$$

provided $x' < x < L$. Eqs. (13) and (14) then imply that

$$\frac{\partial B}{\partial x} = \int_{-\infty}^{\infty} \frac{\partial G}{\partial x}(\mathbf{x},\mathbf{0},t,\tau)\frac{\partial Q}{\partial \tau}(\tau)\,\mathrm{d}\tau \tag{21}$$

which reduces to $\partial Q/\partial t = \partial Q/\partial t$ as $x \to +0$ when use is made of the acoustic relation $\partial U/\partial t = \ \ \partial B/\partial x$, where U is the supraglottal acoustic particle velocity near the glottis, in terms of which $Q = A_2 U$.

3.1. Source filter theory in the absence of reflections

It will be useful in what follows to have available the source filter representation of the sound in the supraglottal tract in the artificial limit in which the reflection of sound from the mouth is suppressed. In this case the observation point $\mathbf{x}$ is within the supraglottal tract (which for the uniform duct model of Fig. 1 may now be assumed to extend over the interval $0 < x < \infty$), and when the supraglottal tract is acoustically closed at the glottis Green's function becomes

$$G(\mathbf{x},\mathbf{y},t,\tau) \approx \frac{c_0}{2A_2}\left\{H\left(t\ \ \tau\ \ \frac{|x\ \ x'|}{c_0}\right) + H\left(t\ \ \tau\ \ \frac{(x+x')}{c_0}\right)\right\}, \quad 0 < x,\ x' < \infty, \tag{22}$$

for sound of wavelength $\gg \sqrt{A_2}$, where $H(\cdot)$ denotes the Heaviside step function.

The dominant 'monopole' component of the sound in the supraglottal tract is now unaffected by supraglottal resonances (the filter being uniformly 'white') and is found from Eqs. (13), (14) and (22) to be

$$p(\mathbf{x},t) \approx \frac{\rho_0 c_0}{A_2} Q\left(t\ \ \frac{x}{c_0}\right), \quad x > 0. \tag{23a}$$

Similarly, in the subglottal tract one has

$$p(\mathbf{x},t) \approx 2p_I\ \ \frac{\rho_0 c_0}{A_1} Q\left(t + \frac{x}{c_0}\right), \quad x < 0. \tag{23b}$$

4. Aerodynamic theory of voicing

The utility of expressions such as (16) and (23a) for the voiced pressure is dependent on an adequate knowledge of the source strength $Q(t)$. This must be determined by consideration of the compressible aerodynamic motion in the whole of the vocal system. To do this for the simplified models of Fig. 1 it is necessary to revert to the coupled system of Fig. 1a. Then the filtering effects of the supraglottal resonances introduce an acoustic back reaction on the glottis that is not necessarily insignificant, but whose inclusion in the equations tends to obscure the simplicity of the derivation of the Fant equation from Eq. (13). Therefore, in the present discussion the influence of acoustic resonances on the glottis source strength $Q(t)$ will be neglected, and we shall actually investigate the system illustrated in Fig. 2a, in which the open end of the supraglottal tract is ignored.

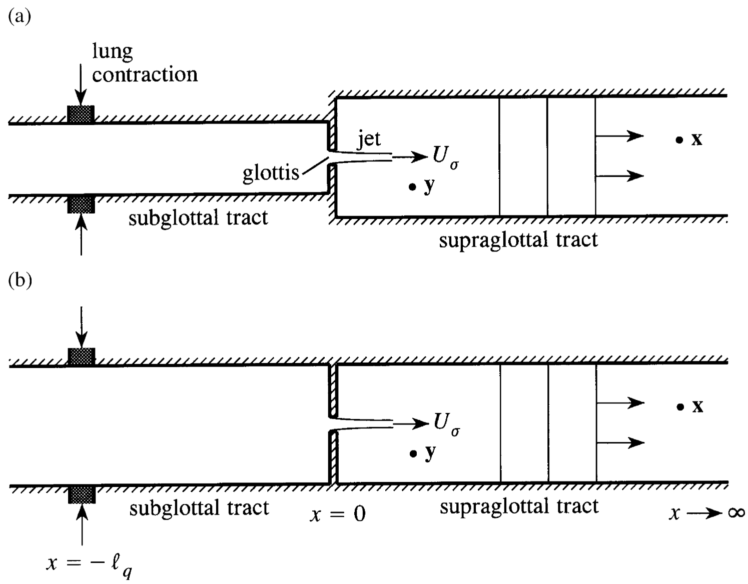

Fig. 2. (a) Aerodynamic components of the theory of voicing for an idealised vocal tract in which reflections from the mouth and lung complex are ignored. (b) Simplified geometry discussed in the text.

Voicing is imagined to be initiated by a nominally uniform subglottal overpressure produced by the steady contraction of the lungs. Acoustic reflection from the lungs will also be ignored, and the overpressure is assumed to be generated in the model of Fig. 2a by an essentially peripheral contraction of a short section of the otherwise uniform subglottal tract that behaves as a volume source of fixed overall strength q, say. Sound generated at the glottis in the subglottal tract is assumed to radiate without hindrance to $x = -\infty$, and this will be taken to be equivalent to the absorption of the sound in the lung complex.

Details of the calculations will be given for the simpler case illustrated in Fig. 2b, wherein it is assumed that the subglottal and supraglottal cross sections are the same: $A_1 = A_2 = A$. This has the advantage of avoiding unnecessary complications in the formulae. Results for the more general case can be derived by an obvious but more involved extension of our approach.

4.1. The overpressure produced by contraction of the lung cavity

Let the peripherally distributed constant volume source representing contraction of the lung cavity be centred at $x = -\ell_q$ and have an axial extent very much smaller than the acoustic wavelengths. Suppose also that the overall source strength q is written in the form

$$q = \frac{2Ap_I}{\rho_0 c_0}, \tag{24}$$

where p_I denotes a constant pressure that will be defined below and, without loss of generality, contraction of the lungs is assumed to begin at time $t = -\ell_q/c_0$. Then the effect of q can be represented by the following distribution of normal velocity on the duct wall:

$$v_n = \frac{2Ap_I}{\ell_p \rho_0 c_0} H\left(t + \frac{\ell_q}{c_0}\right)\delta(x + \ell_q), \tag{25}$$

where v_n is directed into the duct and ℓ_p is the duct perimeter.

The disturbance generated by this source is determined by that part of the surface integral in Eq. (13) involving $\partial \mathbf{v}/\partial \tau$. The component that propagates towards the glottis in Fig. 2b is found by taking $G(\mathbf{x},\mathbf{y},\mathbf{t},\tau)$ to be defined as in Eq. (22) after discarding the second Heaviside function in the brace brackets, and after replacing A_2 by A. Hence, p_I is the amplitude of the overpressure incident on the glottis produced by contraction of the lungs given by

$$p = p_I H\left(t - \frac{x}{c_0}\right). \tag{26}$$

This initiates motion of the glottis (at $x \sim 0$) after its arrival at $t \sim 0$.

4.2. Green's function for the vocal tract

The calculation of the sound radiated into the supraglottal tract must make use of Green's function $G(\mathbf{x},\mathbf{y},t,\tau)$ for the whole system. A generalised formula for G is readily derived in the approximation in which all acoustic wavelengths are large compared with the characteristic diameter $\sim \sqrt{A}$ of the vocal tract (by the method described in [24]). In particular we need its functional form when the observer location $\mathbf{x} = (x,y,z)$ is many duct diameters from the glottis, and for source positions $\mathbf{y} = (x',y',z')$ both near and far from the glottis on the scale of $\sqrt{A}$. These are given by the following formulae in the case where $A_1 = A_2 = A$):

$$G(\mathbf{x},\mathbf{y},t,\tau) = \frac{c_0}{2A} H\left(t - \tau - \frac{|x|}{c_0}\right)\left\{1 + \frac{2\,\mathrm{sgn}(x) Y(\mathbf{y},\tau) \mathrm{e}^{-\int_{\tau}^{t-|x|/c_0} 2c_0\,\mathrm{d}\xi/\ell(\xi)}}{\ell(\tau)}\right\}, \quad |\mathbf{y}| < O\left(\sqrt{A}\right), \tag{27a}$$

$$G(\mathbf{x},\mathbf{y},t,\tau) = \frac{c_0}{2A} H\left(t - \tau - \frac{|x - x'|}{c_0}\right)\{1 - \mathrm{e}^{-\int_{\tau+|x'|/c_0}^{t-|x|/c_0} 2c_0\,\mathrm{d}\xi/\ell(\xi)}\}, \quad |\mathbf{y}| \gg \left(\sqrt{A}\right),\ \mathrm{sgn}(x) \neq \mathrm{sgn}(x'), \tag{27b}$$

$$G(\mathbf{x},\mathbf{y},t,\tau) = \frac{c_0}{2A}\left\{H\left(t - \tau - \frac{|x - x'|}{c_0}\right) + H\left(t - \tau - \frac{|x + x'|}{c_0}\right)\mathrm{e}^{-\int_{\tau+|x'|/c_0}^{t-|x|/c_0} 2c_0\,\mathrm{d}\xi/\ell(\xi)}\right\}, \quad |\mathbf{y}| \gg \left(\sqrt{A}\right),\ \mathrm{sgn}(x) = \mathrm{sgn}(x'), \tag{27c}$$

where in all cases $|\mathbf{x}| \gg \sqrt{A}$.

The function $Y(\mathbf{y},\tau)$ has the dimensions of length and is a solution of Laplace's equation that satisfies $\partial Y/\partial y_n = 0$ on the surface of the vocal tract and on the vocal folds at time τ, where y_n is a local surface normal coordinate. It represents the velocity potential of an irrotational flow through the glottis, and is normalised such that

$$Y \sim x' \pm \frac{\ell(\tau)}{2}, \quad x' \to \pm\infty, \tag{28}$$

where ℓ is the time dependent Rayleigh 'end correction' of the glottis [35], defined by

$$\ell(\tau) = \int_{-\infty}^{\infty} \left(\frac{\partial Y}{\partial x'}(\mathbf{y},\tau) \quad 1 \right) \mathrm{d}x', \tag{29}$$

the integration being along a path through the glottis. The quantity $\rho_0 \ell/A$ corresponds to the 'inductance' $\overline{\ell}$ of Fant's [1] original equation (1), although it is not necessarily numerically the same.

These formulae may now be used in (13) to calculate the pressure wave radiated into the supraglottal tract following the initiation of the overpressure from the lung source (25). Note that $\ell(\tau) \to \infty$ at those source times τ at which the glottis is closed. In particular, for sources near the glottis in $x' > 0$ (such as the jet vorticity), Eqs. (28) and (29) imply that $2Y(\mathbf{y},\tau)/\ell(\tau) \to 1$ as the glottis closes. Then near the wavefront $t \quad x/c_0 \sim \tau$ in $x > 0$ we have

$$\mathrm{e}^{-\int_{\tau}^{t \quad x/c_0} 2c_0 \, \mathrm{d}\xi/\ell(\xi)} \sim 1,$$

and the effective form of the component (27a) of Green's function reduces to the monopole of (22) with $x' = 0$.

A representation of $G(\mathbf{x},\mathbf{y},t,\tau)$ analogous to (27) can in principle be derived when vocal tract resonances are taken into account. Although this would supply a more realistic description of the glottal motion, the resulting increased complexity would obscure important aspects of the glottis flow interaction to be discussed in Section 5.

4.3. Direct radiation produced by lung contraction

This is calculated using Green's function approximation (27b) to calculate the contribution from the lung surface source to the first surface integral of (13). If the corresponding component of the acoustic pressure is denoted by p_q, we find

$$p_q(\mathbf{x},t) \equiv p_q(x,t) = p_I H\left(t \quad \frac{x}{c_0}\right)\{1 \quad \mathrm{e}^{-\int_0^{t \quad x/c_0} 2c_0 \, \mathrm{d}\xi/\ell(\xi)}\}, \quad x \gg +\sqrt{A}. \tag{30}$$

The second term in the brace brackets of this formula is important only near the front $x = c_0 t$ of the wave transmitted into the supraglottal tract, where it determines the smoothing of the incident step wave front (26) produced by passage through the glottis. When ℓ is small, so that the glottis is wide open, it is equivalent to a small shift in the phase of the wavefront, because

$$H\left(t \quad \frac{x}{c_0}\right)\{1 \quad \mathrm{e}^{-\int_0^{t \quad x/c_0} 2c_0 \, \mathrm{d}\xi/\ell(\xi)}\} \sim H\left(t \quad \frac{x}{c_0}\right) \quad \frac{\ell(0)}{2c_0}\delta\left(t \quad \frac{x}{c_0}\right) \sim H\left(t \quad \frac{x}{c_0} \quad \frac{\ell(0)}{2c_0}\right).$$

4.4. Vortex sound contribution

The supraglottal radiation produced by the Lamb vector source $\boldsymbol{\omega} \wedge \mathbf{v}$ in the volume integral on the right of (13) is dominated by vorticity in the immediate neighbourhood of the glottis, where it behaves as a dipole source associated with the unsteady drag on the vocal folds. Vorticity in the flow downstream of the glottis produces sound of much weaker quadrupole strength. Thus, Green's function approximation (27a) must be used to evaluate the final integral on the right of (13).

In principle $\boldsymbol{\omega} \wedge \mathbf{v}$ can be evaluated to any degree of approximation provided the space time distributions of velocity and vorticity are known near the glottis. These might be derived, for example, from detailed observation or from a preliminary numerical investigation of the glottal flow. In the absence of such data, however, a simple analytical estimate of the vorticity contribution, which is consistent with previous treatments of the influence of the jet flow from the glottis, can be obtained (following Howe and McGowan [25]) by assuming that the jet vorticity is confined to a free shear layer of infinitesimal thickness (i.e. a vortex sheet) at the edge of the jet emerging from the glottis (see Fig. 3). The vortex sheet has circulation U_σ per unit tangential length of the sheet, where U_σ is the jet speed just inside the shear layer, and the vorticity is convected by the flow at speed $\frac{1}{2}U_\sigma$. Then we can put $\boldsymbol{\omega} \wedge \mathbf{v} = \frac{1}{2}U_\sigma^2 \delta(s_\perp)\mathbf{n}$, where $s_\perp$ is the distance measured from the vortex sheet in the direction of the jet outward unit normal $\mathbf{n}$ (Fig. 3), so that

$$\int_{V(\tau)} \frac{\partial Y}{\partial \mathbf{y}} \cdot (\boldsymbol{\omega} \wedge \mathbf{v})(\mathbf{y},\tau) d^3\mathbf{y} \approx \frac{1}{2} \oint_{S_j(\tau)} \frac{\partial Y}{\partial y_n} U_\sigma^2(\mathbf{y},\tau) \, \mathrm{d}S(\mathbf{y}) \tag{31}$$

in the final integral of (13). The integral on the right is taken over the surface $S_j(\tau)$ of the vortex sheet enclosing the jet. It can be evaluated by reference to Fig. 3, which illustrates the instantaneous family of streamsurfaces of a hypothetical flow through the glottis defined by the potential function $Y(\mathbf{y},\tau)$. The main contribution to the surface integral is from that section of the jet just outside the glottis, where the jet shear layer is intersected by the Y streamlines. The length of jet involved is comparable to the diameter of the glottis, and is much smaller than the effective radius h, say, of the vocal tract. Thus, if f_0 is the characteristic frequency of the vocal folds, it may be assumed that U_σ is independent of $\mathbf{y}$ within the integrand provided that $f_0 h/U_\sigma \ll 1$. Because this condition is usually satisfied we can put

$$\int_{V(\tau)} \frac{\partial Y}{\partial \mathbf{y}} \cdot (\boldsymbol{\omega} \wedge \mathbf{v})(\mathbf{y},\tau) \, \mathrm{d}^3\mathbf{y} \approx \frac{1}{2} U_\sigma^2(\tau) \oint_{S_j(\tau)} \frac{\partial Y}{\partial y_n} \, \mathrm{d}S(\mathbf{y}) \equiv \frac{AU_\sigma^2(\tau)}{2}, \tag{32}$$

where the final integral has been evaluated using the normalisation condition (28).

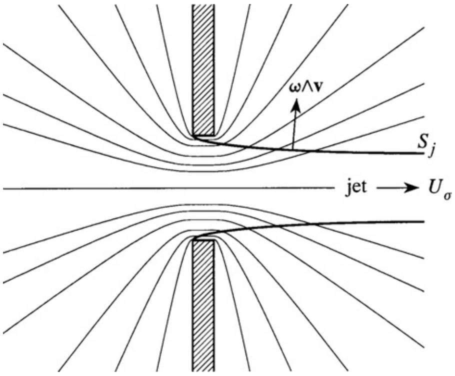

Fig. 3. Intersection of the streamsurfaces of the potential function $Y(\mathbf{y},\tau)$ and the jet shear layer, within which $\boldsymbol{\omega}\wedge\mathbf{v}$ is in the direction of the outward normal to the jet.

It now follows from (27a) and (13) that the net contribution to the supraglottal pressure from the jet vorticity p_σ, say, can be cast in the form

$$p_\sigma(x,t) \approx -\frac{\rho_0 c_0}{2}\int_{-\infty}^{t-x/c_0} \frac{U_\sigma^2(\tau)\mathrm{e}^{-\int_\tau^{t-x/c_0} 2c_0\,\mathrm{d}\xi/\ell(\xi)}}{\ell(\tau)}\,\mathrm{d}\tau. \tag{33}$$

4.5. Surface sources within the glottis

We have already considered the contribution from the lung contraction source (25) to the surface integral in the representation (13) of the supraglottal sound. Within the confines of our rigid wall model of the vocal tract only the moving walls of the glottis can supply an additional contribution to this integral when viscous losses at the boundaries are ignored. This is usually interpreted as a small amplitude acoustic monopole associated with volumetric changes that occur during vibration of the vocal folds. For a more realistic model of the glottis, however, it appears that tissue flexure when the folds touch can also generate a weak monopole that is significant just after and during glottal closure [36].

But, there is another dipole contribution whose presence can be understood by recalling that $\rho_0\partial\mathbf{v}/\partial t$ determines the rate of change of fluid momentum at a fixed position — in fact, it is precisely a surface integral of this kind that determines the excess drag force experienced by a body whose cross section relative to a mean flow varies with time [37]. Thus far our analysis has proceeded without specific reference to the structural details of the glottis. But the contribution from this source depends on glottis geometry. To illustrate its likely importance we shall henceforth confine the discussion to a clean, mechanical model of the vocal folds introduced by Howe and McGowan [30].

5. The glottal surface source

5.1. Single mass model of the vocal folds

We adopt a simple model in which the vocal tract near the glottis is assumed to have rectangular cross section of span $\ell_3 = 2a$, width $2h$, and cross sectional area A=$4ah$, as indicated in Fig. 4, which shows a cross section in a median plane parallel to the mean flow, the mean flow being from left to right (cf. [30,31]). The coordinate origin is at the centre point O of the glottis; the y and z axes are respectively parallel to h and out of the plane of the paper in Fig. 4. The two dimensional folds of thickness $\ell_g \ll h$ extend over the full span ($|z|<a$). The volume flux through the glottis is modulated by fluctuations in the glottal cross sectional area, which resembles more or less an ellipse of large aspect ratio. This elongated shape is approximated by a rectangle whose longer side has fixed length $2a$ and has time dependent width $2\zeta(t)$ (Fig. 5).

The oscillations of the glottal width $2\zeta(t)$ (Fig. 4) are taken to satisfy

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2}+\gamma_0\frac{\mathrm{d}\zeta}{\mathrm{d}t}+\Omega^2(\zeta-\zeta_0) = -\frac{F(t)}{m}, \tag{34}$$

where m is the effective mass of each fold, ζ_0 is the equilibrium semi width of the glottis, γ_0 is a structural damping coefficient, $\Omega = 2\pi f_0$ is a radian natural frequency determined by muscular adjustment of the folds, and $F(t)$ is the force exerted on the glottal wall.

Changes in $\zeta(t)$ are driven by the flow through the glottis, which involves separation from the glottal wall and jet formation. The separation point changes during the inward and outward motion of the folds in the manner illustrated in Fig. 4. In condition (a) $\mathrm{d}\zeta/\mathrm{d}t > 0$ and the glottis is expanding: the incoming flow separates at the local 'trailing edge' at the front edge A of the glottis, so that the pressure applied over the glottal walls is uniform and approximately the same as the locally uniform supraglottal pressure. In case (b) $\mathrm{d}\zeta/\mathrm{d}t \le 0$, the glottal area is decreasing and vorticity is now shed from the neighbourhood of the trailing edge at B.

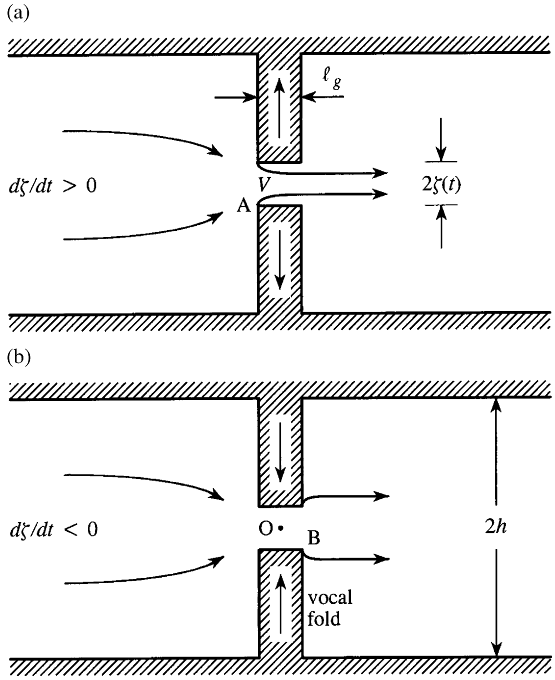

Fig. 4. 'Single-mass' model of the glottis: (a) vortex shedding occurs from the leading edge of an expanding glottis; (b) shedding occurs from the aft edge during contraction.

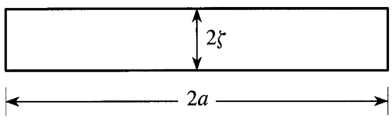

Fig. 5. Rectangular approximation to the shape of the glottal cross-section during phonation.

The Strouhal number of motion within the glottis $f_0\ell_g/V \sim 0.1$, where the mean flow speed in the glottis $V \sim 20$ m/s and ℓ_g is the streamwise length of the glottis. This is small enough for it to be assumed that, over a cycle of oscillation at frequency f_0 the fluid passes through a continuous sequence of quasi static states. The motion is at sufficiently high Reynolds number that the direct action of viscosity within the flow can be neglected. Pressure forces on the folds are then confined to the walls of the glottis, i.e. to the end faces of the rectangular fold models. These end forces are negligible when the glottis expands; during contraction the end faces experience the suction force $F(t)$. This was calculated by Howe and McGowan [30] in the limit of small aspect ratio ℓ_g/h by neglecting the fold finite thickness ℓ_g, replacing each rectangular fold by a thin rigid *blade* (coinciding with the y axis in Fig. 6), and using free streamline theory to calculate the separated flow from the vicinity of the blade tip [37 41].

During expansion of the glottis ($\mathrm{d}\zeta/\mathrm{d}t > 0$) separation occurs at the subglottal 'leading edge' A of the blade shown in Fig. 6, the corresponding vortex sheet (free streamline) of the 'lower' half of the two dimensional jet being indicated by the broken line curve in the figure. In the contraction phase the free streamline is shed from the point B on the supraglottal face ($x=+0$) of the fold, which is a short distance below the tip (too small to be shown clearly in the figure). The free streamline forms the quasi static edge of the jet. The asymptotic jet *half width* $s(t)$ is attained within a distance of about $2\zeta(t)$ from the glottis, where the jet velocity becomes uniform and equal to $U_\sigma = V/\sigma$. When $\zeta \ll h$ we find [30]

$$\sigma \equiv \frac{s(t)}{\zeta(t)} \approx \begin{cases} 0.62, & \mathrm{d}\zeta/\mathrm{d}t > 0, \\ 1.15, & \mathrm{d}\zeta/\mathrm{d}t < 0. \end{cases} \tag{35}$$

The suction force F is exerted on the glottal wall when $\mathrm{d}\zeta/\mathrm{d}t \le 0$ (Figs. 4b and 6) and is otherwise zero:

$$F = \begin{cases} \dfrac{1}{3^{3/2}\pi}\rho_0 U_\sigma^2 \sigma \Delta, & \dfrac{\mathrm{d}\zeta}{\mathrm{d}t} \le 0 \text{ and } \zeta > 0, \\ 0, & \dfrac{\mathrm{d}\zeta}{\mathrm{d}t} > 0 \text{ or } \zeta \le 0, \end{cases} \tag{36}$$

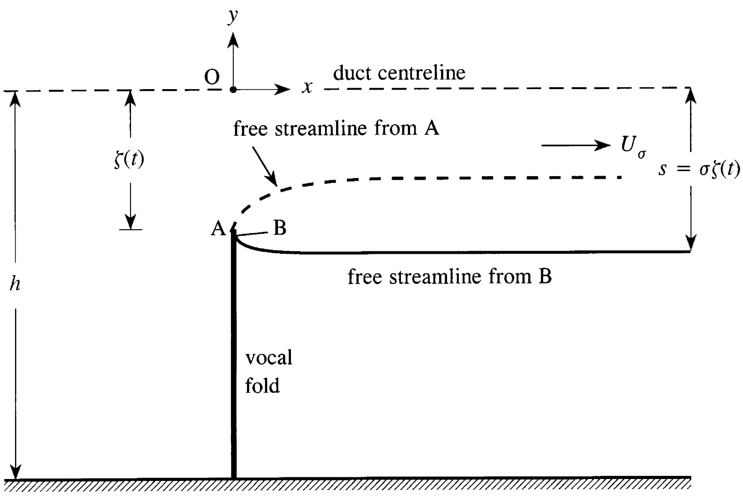

Fig. 6. Free streamline model used by Howe and McGowan [30] to calculate the separated flow through the glottis. The separation point B on the supraglottal face $x \quad +0$ of the blade is a small distance below the tip.

where $\Delta \equiv \Delta(t) = 4a\zeta(t)$ is the cross sectional area of the glottis. This suction force is necessary to pull the potential flow around the tip of the blade, and is an idealised illustration of the *Coanda effect* (Fant's Bernoulli suction [1]), where convergence of the flow streamlines near the blade tip results in a dramatic fall in pressure [37].

For this thin blade model the time $\sim \ell_g/V$ required for the separation point to move between the effective 'ends' A and B of the glottis is negligible. A consequence of this is the appearance of small discontinuities in the predicted waveform of the supraglottal sound. Similar discontinuities were avoided by Zanartu et al. [6] by arbitrarily smoothing abrupt changes in the jet 'discharge coefficient'.

Impact of the vocal folds occurs when $\zeta \to 0$. Two alternative modes of operation can be examined when solving Eq. (34): (i) the impact is *inelastic*, so that $\mathrm{d}\zeta/\mathrm{d}t \to 0$ when $\zeta = 0$; (ii) impact is avoided by permitting the blade tips to overlap, i.e. the blades complete their full oscillations determined by Eq. (34) with ζ allowed to assume negative values (when $F \equiv 0$). In case (i) motion of the folds after impact is initiated by elastic forces; in case (ii) the glottis is *closed* for the finite time during which the blades overlap. When $\zeta > 0$ the corresponding thin blade 'end correction' $\ell(t)$ of Eq. (28) is known in analytic form [25,42]. When, in particular, $\zeta \ll a$

$$\ell(t) \approx \frac{A}{\pi a} \ln\left(\frac{2h}{\pi\zeta(t)}\right). \tag{37}$$

5.2. Sound produced by glottal surface sources

To calculate the component p_g of the supraglottal pressure generated by the glottal surface source it is necessary to have an analytic representation of the separated flow near the blade tip during contraction of the glottis. This is found by expressing the flow velocity in terms of the complex variable $\mathcal{Z} = x + \mathrm{i}y$, where x, y are defined as in Fig. 6. Then the blade tip in Fig. 6 corresponds to the point $\mathcal{Z} = \quad \mathrm{i}\zeta$.

In the neighbourhood of the tip, when the glottis is contracting and separation occurs from the supraglottal face of the vocal fold, the complex potential $w(\mathcal{Z})$ of the flow within the jet is found to be [30]

$$w \sim U_\sigma \left(\frac{8\mathrm{i}s}{3^{3/2}\pi}\right)^{1/2} \sqrt{\mathcal{Z} + \mathrm{i}\zeta}. \tag{38}$$

This square root singularity at the blade tip is responsible for the Coanda suction force $F(t)$.

This velocity singularity also occurs in the integrand of the first surface integral of (13) that defines p_g. In addition, however, there is a similar singularity in the integrand associated with the behaviour of the potential function $Y(\mathbf{y},\tau)$ at the blade tip. A simple calculation using conformal mapping reveals that near the tip $\mathcal{Z} = \quad \mathrm{i}\zeta$ of Fig. 6 the singular part of $Y(\mathbf{y},\tau)$ is given by

$$Y(\mathbf{y},\tau) \sim \mathrm{Re}(w_Y(\mathcal{Z})), \quad w_Y(\mathcal{Z}) = \frac{A}{\pi a}\left(\frac{\mathrm{i}}{2\zeta}\right)^{1/2} \sqrt{\mathcal{Z} + \mathrm{i}\zeta}. \tag{39}$$

These formulae are now used to evaluate p_g from the following component of (13):

$$p_g(\mathbf{x},t) = \rho_0 \int_{-\infty}^{\infty} \oint_{S_g(\tau)} G \frac{\partial \mathbf{v}}{\partial \tau} \cdot \mathrm{d}\mathbf{S}(\mathbf{y})\, \mathrm{d}\tau, \tag{40}$$

where the surface integral is confined to the moving surface S_g of the glottal region, and G is given by (27a).

To avoid having to evaluate $\partial \mathbf{v}/\partial\tau$ on the moving surface, we first make the transformation

$$\oint_{S_g(\tau)} G\frac{\partial \mathbf{v}}{\partial \tau}\cdot \mathrm{d}\mathbf{S}(\mathbf{y}) = \frac{\partial}{\partial \tau}\oint_{S_g(\tau)} G v_n\, \mathrm{d}S(\mathbf{y}) \quad \oint_{S_g(\tau)}\left(\frac{\partial G}{\partial \tau}+\frac{\partial (v_j G)}{\partial y_j}\right) v_n\, \mathrm{d}S(\mathbf{y}), \tag{41}$$

where v_n is the normal component of velocity on $S_g(\tau)$ directed into the fluid. The first term on the right integrates to zero when used in Eq. (13). In the thin blade approximation to the vocal folds v_n=0 on $S_g(\tau)$ except at the blade tips, where $v_n = \quad \partial\zeta/\partial\tau$. This would normally imply that the remaining surface integrals on the right of (41) are null, because of the infinitesimal glottal width ($\ell_g \to 0$). However, according to Eq. (39) $\partial Y(\mathbf{y},\tau)/\partial y_j$ is singular at the blade tips; similarly, Eq. (38) shows that the flow velocity is also singular there provided the glottis is *contracting*. In Fig. 6 their complex product has a simple pole at $\mathcal{Z} = \quad i\zeta$. This singularity is sufficient to render a non zero value for the component of the second integral in (41) involving $\partial G/\partial y_j$. The integral can be evaluated by residues or by the method described by Batchelor [40, Section 6.5] yielding

$$p_g(\mathbf{x},t) \equiv p_g(x,t) \approx \frac{4\rho_0 c_0}{\sqrt{\pi}3^{3/2}}\int_{-\infty}^{t-x/c_0} H(\quad \dot\zeta)\dot\zeta\, \frac{\sigma^{1/2}U_\sigma \mathrm{e}^{-\int_\tau^{t\quad x/c_0} 2c_0\, \mathrm{d}\xi/\ell(\xi)}}{\ell(\tau)}\, \mathrm{d}\tau, \tag{42}$$

where $\dot\zeta = \partial\zeta/\partial\tau$, and the Heaviside function indicates that this component of the sound is generated only when the glottis contracts, when the mean flow wets the walls of the glottis.

It may be noted that the singular term responsible for p_g in the second integrand on the right of Eq. (41) is related to the transverse transport of streamwise fluid momentum induced by the blade motions, i.e. to the action of Reynolds stresses. This produces a time dependent drag force and dipole radiation.

6. The generalised Fant equation

The net acoustic pressure radiated into the supraglottal tract is given by

$$p(x,t) = p_q(x,t) + p_\sigma(x,t) + p_g(x,t), \tag{43}$$

where the components on the right are given respectively by Eqs. (30), (33) and (42).

Let $U(t) \equiv Q/A$ be the limiting value of the acoustic particle velocity $p(x,t)/\rho_0 c_0$ as the glottis is approached from $x>0$, i.e. put

$$U(t) = \lim_{x\to+0}\frac{p(x,t)}{\rho_0 c_0}. \tag{44}$$

The formal limit of Eq. (43) using formulae (30), (33) and (42) then yields the required integral equation representation of the generalised Fant equation (3). Following Howe and McGowan [25] we can then show (using Eq. (44)) by differentiation of this equation that

$$\ell(t)\frac{\partial U}{\partial t} + 2c_0 U = \frac{2p_I}{\rho_0}H(t) \quad \frac{U_\sigma^2}{2} + 4\sqrt{\frac{\sigma}{\pi 3^{3/2}}}H(\quad \dot\zeta)\frac{\partial \zeta}{\partial t}U_\sigma, \tag{45}$$

where on the right hand side $U_\sigma \equiv U_\sigma(t)$, $\zeta \equiv \zeta(t)$.

By continuity $U_\sigma = hU/\sigma\zeta$. Hence the generalised Fant equation (3) reduces to

$$\ell(t)\frac{\partial U}{\partial t} + 2c_0 U + \left(\frac{h}{\sigma\zeta}\right)^2\frac{U^2}{2} \quad \frac{4hH(\quad \dot\zeta)}{\sqrt{\sigma\pi 3^{3/2}}}\frac{\partial\zeta}{\partial t}\frac{U}{\zeta} = \frac{2p_I}{\rho_0}H(t), \tag{46}$$

a result that can also be expressed in the form (because $\partial\ell/\partial t = \quad 4h\dot\zeta/\pi\zeta$)

$$\ell(t)\frac{\partial U}{\partial t} + 2c_0 U + \left(\frac{h}{\sigma\zeta}\right)^2\frac{U^2}{2} + \sqrt{\frac{\pi}{\sigma 3^{3/2}}}H(\quad \dot\zeta)U\frac{\partial \ell}{\partial t} = \frac{2p_I}{\rho_0}H(t). \tag{47}$$

The three terms on the right of Eq. (45) are sources associated respectively with the lung overpressure, jet vorticity, and the Reynolds stress drag in the glottis. The latter two terms are both sources of ‘expansion’ that modulate the strength of the lung overpressure transmitted into the supraglottal tract. All of these sources are balanced against the terms on the left hand side of the equation representing fluid inertia in the glottis ($\ell\partial U/\partial t$) and its forcing by the radiated sound pressures on each side of the glottis ($2c_0U$). Because $f_0 h/c_0 \ll 1$, inertia and the Reynolds stress source tend to be small over most of the voice cycle. When they are neglected the equation reduces to a ‘quasi static’ quadratic equation for U [30]. On the other hand, the radiation term $2c_0U$ is always finite and non negligible during the open period of the glottal cycle, because the Mach number U/c_0 is small. Thus, source tract coupling represented by this term cannot be ignored. The analytical form of this coupling is determined by the specific form of the acoustic Green's function (27) used in the general representation (13) of the acoustic field. The procedure described above to derive the Fant equation from Eq. (13) is applicable also when the influence of acoustic resonances of the vocal tract are taken into account. In that case the back reaction of the acoustic field on the glottal motion is determined by the functional form of the corresponding modified Green's function $G(\mathbf{x},\mathbf{y},t,\tau)$.

When $U(t)$ is known from the solution of (47) the radiation from the glottis in both the supraglottal and subglottal tracts is more conveniently expressed in the respective forms

$$p(x,t) = \begin{cases} \rho_0 c_0\, U\left(t - \dfrac{x}{c_0}\right), & x \gg +\sqrt{A} \text{ (supraglottal)}, \\ p_I\left\{H\left(t - \dfrac{x}{c_0}\right) + H\left(t + \dfrac{x}{c_0}\right)\right\} - \rho_0 c_0\, U\left(t + \dfrac{x}{c_0}\right), & x \ll -\sqrt{A} \text{ (subglottal)}. \end{cases} \tag{48a, b}$$

The second line of (48a, b), giving the radiation into the subglottal region, can be derived using Green's function (27) applied to the region $x<0$. The term involving the incident pressure amplitude p_I reduces to the absolute constant $2p_I$ when $t>|x|/c_0$; it then represents a uniform pressure field with no associated mean flow; all of the volume flux is then furnished by the glottal volume flux acoustic particle velocity U.

6.1. Continuation of the solution through a sustained closure of the glottis

Fant's equation (47) has been derived using Green's function (27) under the implicit assumptions that: (i) voicing is initiated by the normal velocity distribution (25) on the lung periphery, and (ii) $\zeta>0$, so that the glottis remains 'open', and indeed (as pointed out by a referee) phonation in practice often involves some minimal level of 'leakage', even when the glottis is nominally closed. However, the glottis does close in certain circumstances (in 'pressed' voice, for example) so that the analytical consequences of complete closure during each cycle of period $\sim 1/f_0$ must be examined. The volume flux Q and associated near field acoustic particle velocity $U(t)$ both vanish when the glottis is closed. The supraglottal acoustic near field is then null, but there remains a constant elevated pressure $p=2p_I$ near the glottis in the subglottal tract. To determine U after such a period of closure it is necessary to know both the subsequent structure of the Fant equation and the initial conditions that govern its solution.

To fix ideas, suppose the glottis opens at time $t=\tau_0>0$ after a period of closure. Then initially, close to the glottis

$$B = \frac{2p_I}{\rho_0} H(-x) \text{ i.e. } B = \frac{2p_I}{\rho_0},\ x<0, \quad B=0,\ x>0. \tag{49}$$

There is no contribution from the lung surface velocity distribution (25) to the surface integral in the representation (13) of the sound, because $\partial v_n/\partial t = 0$ when $t \geq \tau_0$. The formal contributions from the remaining source terms in (13) are unchanged, however, and the net acoustic pressure in $x>0$ produced after the glottis re opens can be cast in the form

$$p(x,t) = p_{\tau_0}(x,t) + p_\sigma(x,t) + p_g(x,t), \quad t>\tau_0, \tag{50}$$

where p_σ, p_g are defined as in (43), and p_{τ_0} is the pressure field attributable to the initial condition (49).

To determine $p(x,t)$ in $x>0$ as the solution of an initial value problem $B(\mathbf{x},t)$ is replaced by $H(t-\tau_0)B(\mathbf{x},t)$ in the acoustic equation (8). Multiplication of (8) by $H \equiv H(t-\tau_0)$ and rearrangement yields

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_j^2}\right)(HB) = \operatorname{div}(H\boldsymbol{\omega}\wedge\mathbf{v}) + \frac{1}{c_0^2}\left\{\delta(t-\tau_0)\frac{\partial B}{\partial t} + \frac{\partial}{\partial t}(\delta(t-\tau_0)B)\right\}. \tag{51}$$

The causal solution HB of this equation vanishes for $t<\tau_0$ and coincides with $B(\mathbf{x},t)$ for $t>\tau_0$. The second term on the right hand side accounts for the initial discontinuous pressure distribution across the opening glottis. The inertia of the stationary system at the instant $t=\tau_0$ of the opening of the glottis implies (from (49)) that

$$B(\mathbf{x},\tau_0) = \frac{2p_I}{\rho_0}H(-x), \quad \left(\frac{\partial B}{\partial t}\right)_{t=\tau_0} = 0. \tag{52}$$

Therefore, only the second term in the brace brackets on the right of (51) is relevant. The corresponding contribution p_{τ_0} to the acoustic pressure in $x>0$ is obtained by convolution with the approximation (27b) of Green's function. The result can be expressed in the form

$$p_{\tau_0}(x,t) = 2p_I \int_{-\infty}^{\infty} H\left(t-\tau_0-\frac{x}{c_0}+\frac{x'}{c_0}\right)H(-x')\frac{\mathrm{e}^{-\int_{\tau_0-x'/c_0}^{t-x/c_0} 2c_0\,\mathrm{d}\xi/\ell(\xi)}}{\ell(\tau_0-x'/c_0)}\,\mathrm{d}x', \quad x>0,\ t>\tau_0. \tag{53}$$

If we now introduce the corresponding component $U_{\tau_0} = \lim_{x\to+0} p_{\tau_0}/\rho_0 c_0$ of the acoustic particle velocity U, then it is readily deduced by the differentiation of (53) that

$$\ell(t)\frac{\partial U_{\tau_0}}{\partial t} + 2c_0 U_{\tau_0} = \frac{2p_I}{\rho_0}H(t-\tau_0). \tag{54}$$

It now follows as before, by use of the corresponding formulae (33), (42) for p_σ, p_g, that the overall acoustic particle velocity $U(t)$ just inside the supraglottal tract satisfies for $t>\tau_0$ Eq. (45) with the first term on the right hand side replaced by the right hand side of (54). Evidently these terms are equivalent for $t>\tau_0$, and we therefore conclude that the original equation (45) derived only for $\zeta > 0$ remains applicable for all times except during those intervals where the glottis is closed and $U \equiv 0$.

An obvious generalisation of the above argument is applicable also in cases involving a more realistic model of the glottal region where tissue flexure results in the continued production of monopole sound for a short time after closure, or just prior to opening (see, [36]).

7. Numerical results

The prediction (48a) of the voiced sound pressure $p(x,t)$ within the supraglottal tract radiating towards the mouth is determined by the simultaneous solution of the generalised Fant equation (47) and the equation of motion (34) of the single mass model of the vocal folds. Illustrative numerical results will be given in this section for cases where the structural motions are defined by

$$f_0 = 100\ \text{Hz}, \quad \gamma_0/\Omega = 0.01, \quad h = a = 10\ \text{mm}, \quad m = 0.5 \times 10^{-4}\ \text{kg}. \tag{55}$$

7.1. Inelastic fold collisions

The differential equations are integrated using a fourth order *Runge Kutta* procedure. The fold motion and supraglottal sound are excited by the arrival at time t=0 of the lung step pressure rise (26) at the glottis (x=0), subject to the initial conditions

$$U = 0, \quad \zeta = \zeta_0, \quad \frac{\mathrm{d}\zeta}{\mathrm{d}t} = 0 \ \text{ at } t = 0, \tag{56}$$

where $\zeta_0 > 0$ is the undisturbed displacement of each fold. The simple single mass model is self excited from rest by the starting flow through the undisturbed glottis, which is necessarily attached, irrotational and immediately exerts a suction force that initiates movement of the folds.

Fig. 7a shows three cycles of the relative pressure $p/p_I \equiv \rho_0 c_0 U/p_I$ plotted as a function of $f_0 t$ at a fixed point $x > 0$ when $\zeta_0/h = 0.05$ and $p_I = 12$ cm of water. The corresponding variations in ζ/h are displayed in Fig. 7b. The folds are assumed to collide inelastically, by imposition of the condition $\mathrm{d}\zeta/\mathrm{d}t = 0$ when $\zeta = 0$. A consequence of this nonlinear condition on the fold motion, evident from the figure, is a marginal *increase* in the effective natural frequency of the voice pulses above the nominal value f=f_0. A separate nonlinearity is responsible for the rapid variation exhibited by the pressure signature near its peak levels. This is a result of the expansion in the cross sectional area of the glottis jet occurring when the flow becomes attached to the glottis interior 'wall' and the separation point is displaced to the anterior face of the blade like fold (from B in Fig. 6). The displacement of the separation point is instantaneous according to the free streamline approximation of Fig. 6. In reality the transition occurs over a finite relaxation time that causes a broadening of the time interval occupied by the pressure increase. The rapid change in pressure can be removed by introducing an artificial and arbitrary smoothing of the motion of the separation point of the jet (the method adopted in [6], for example). For these results the undisturbed pressure wave p_I transmitted through the glottis is modulated predominately by the nonlinear vortex induced pressure $p_\sigma \sim O(p_I)$. By contrast the edge dipole pressure p_g is an order of magnitude smaller.

A similar behaviour at much reduced acoustic amplitude is depicted in Fig. 8, where p_I is unchanged at 12 cm of water, but ζ_0/h takes the reduced and more reasonable value of 0.01. In this case the inelastic collisions prevent the glottis cross section from becoming large, so that the voiced pulse amplitude remains relatively small.

7.2. Sustained closure of the glottis

The glottis closes when $\zeta \to 0$. Sustained periods of closure occur if the folds collide elastically. This is mimicked in the thin blade, single mass model by permitting the blades to overlap when ζ becomes negative. It may also be assumed that tissue compression occurs when $\zeta < 0$, producing large increases in damping and restoring force in the vocal fold equation (34), but this refinement is not pursued here.

The glottis volume velocity vanishes at retarded times of closure (when $\zeta < 0$) producing corresponding nulls in the supraglottal pressure. Typical predictions are exhibited in Figs. 9 and 10, respectively for $\zeta_0/h = 0.05$, 0.01, and for $p_I = 12$ cm of water. The numerical solution of the Fant equation (47) is restricted to the intervals during which $\zeta > 0$, starting with U=0 when ζ increases through zero; at other times $U \equiv 0$. This is consistent with the predicted continuous variation of $p/p_I \equiv \rho_0 c_0 U/p_I$ as $\zeta \to +0$ shown in the figures.

In both of Figs. 9 and 10 it is evident that the voice pulse frequency is equal to the assumed natural frequency f_0 of the fold oscillations. This is because of the absence of the nonlinear 'inelastic' constraint ($\mathrm{d}\zeta/\mathrm{d}t = 0$ when $\zeta = 0$) on the motion of the vocal folds. The maximum amplitude of the glottal oscillations is now determined by a balance between the nonlinear suction force (35) and the assumed tissue elastic forces. Thus in Fig. 10, where $\zeta_0/h = 0.01$, ζ/ζ_0 attains a maximum value ~ 8.8 (the maximum glottal open area is only marginally smaller than in Fig. 7 where $\zeta_0/h = 0.05$ but fold collisions are inelastic). Consequently, the peak acoustic pressure remains relatively large even when ζ_0/h is small.

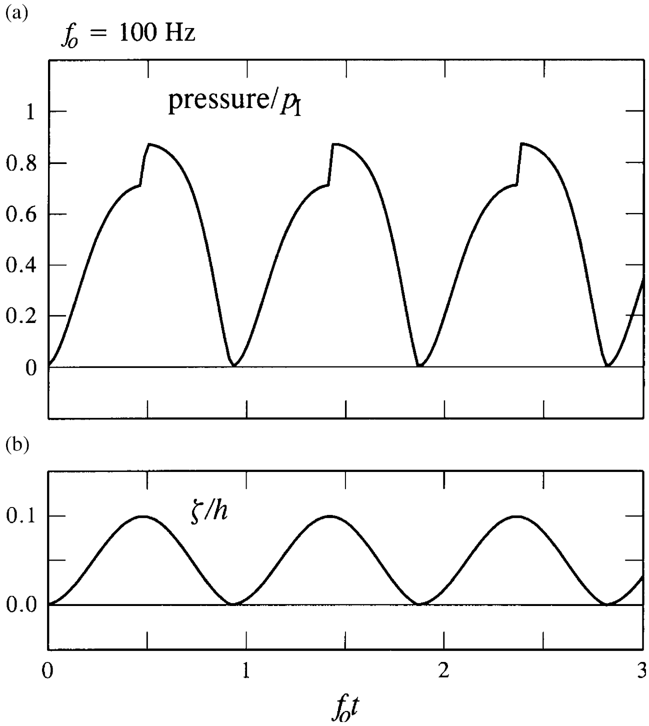

Fig. 7. Inelastic collision of the vocal folds: (a) three cycles of the supraglottal sound pressure predicted by Eq. (48a) for conditions (55) when f_0=100 Hz, ζ_0/h 0.05 and p_I 12 cm of water; (b) the corresponding variation of vocal fold amplitude $\zeta(t)/h$.

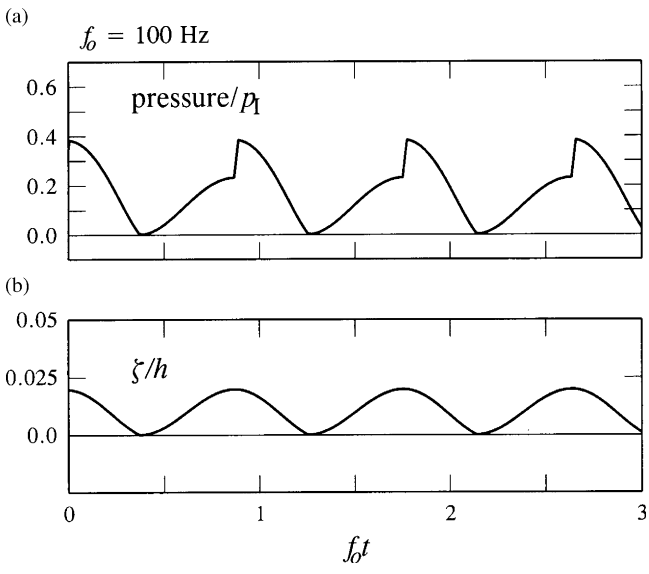

Fig. 8. Inelastic collision of the vocal folds: (a) three cycles of the supraglottal sound pressure predicted by Eq. (48a) for conditions (55) when f_0=100 Hz, ζ_0/h 0.01 and p_I 12 cm of water; (b) the corresponding variation of vocal fold amplitude $\zeta(t)/h$.

8. Conclusion

Voiced speech is the aerodynamic sound produced by the quasi periodic throttling by the glottis of nominally steady flow from the lungs. The fluid structure interactions involved in sound production can be investigated theoretically on several different levels of sophistication. It is now possible, for example, to attempt a full numerical simulation that includes a more or less precise representation of the mechanics of the sound and unsteady air flow (subject to certain compromises regarding turbulence modelling) and a model dependent representation of tissue mechanics. Such

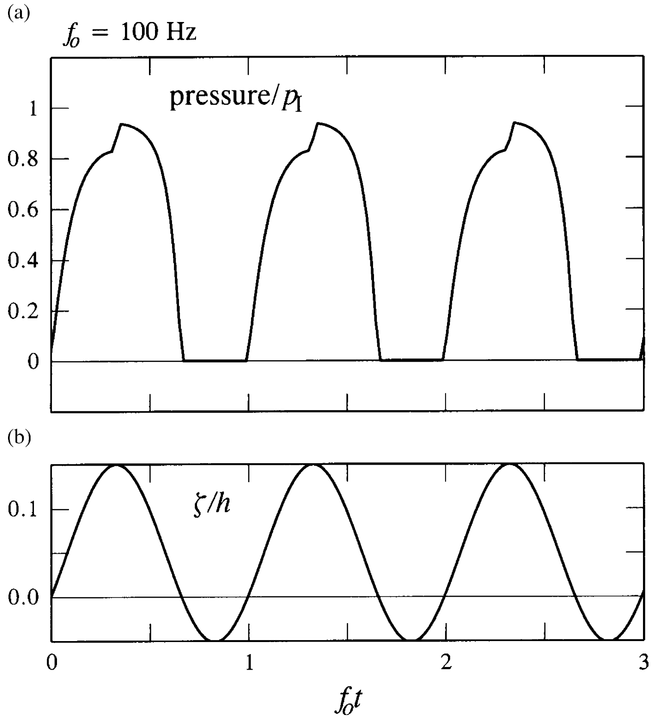

Fig. 9. Influence of glottal closure: (a) three cycles of the supraglottal sound pressure predicted by (48a) for conditions (55) when f_0=100 Hz, ζ_0/h 0.05 and p_I 12 cm of water; (b) the corresponding variation of vocal fold amplitude $\zeta(t)/h$.

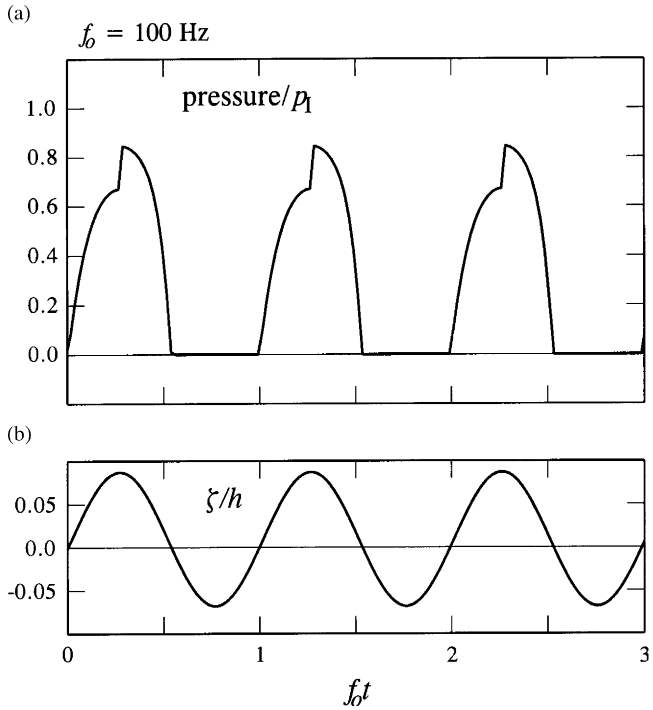

Fig. 10. Influence of glottal closure: (a) three cycles of the supraglottal sound pressure predicted by (48a) for conditions (55) when f_0=100 Hz, ζ_0/h 0.01 and p_I 12 cm of water; (b) the corresponding variation of vocal fold amplitude $\zeta(t)/h$.

procedures are time consuming and limited in extent to one or two voicing cycles. More approximate schemes condense the influence of the unsteady aerodynamics of the interaction into some form of Fant's [1] 'reduced complexity' equation for the glottis volume velocity Q.

It is important that the radiation pressure in the glottal region be included in any analysis of the flow in the glottis; that is, source tract interaction is crucial to a proper understanding of glottal flow. It provides an essential component of the reduced complexity differential equation, and is significant even in the absence of resonances in the vocal tract. Fant's original theory was based on heuristic modelling of the flow and the classical 'lumped element' electrical analogy frequently adopted in one dimensional acoustics. Our aim in this paper has been to present a systematic and clear derivation of the general form of Fant's equation (3) based on the application of the equations of aerodynamic sound. Such a procedure can in principle be performed to a degree of approximation that depends only on the accuracy with which the time varying flow geometry and surface acoustic boundary conditions can be specified. The advantage of our method is that it can handle all of the effective 'source terms' governing Q and determines the manner in which they contribute to the generalised Fant equation; the equation itself cannot provide details of the hydrodynamic flow everywhere within the vocal tract, but will supply a description of the acoustic properties that is consistent with the level of understanding of the main features of the flow surface interactions within the system. The procedure has been illustrated in this paper by consideration of a simplified, self sustaining single mass model of the vocal folds. This uses free streamline theory to incorporate the joint actions of surface friction and flow separation at the glottis. A new source has been identified in the Fant equation that is related to the Reynolds stress contribution to the vocal fold drag, associated with their oscillatory motion transverse to the mean flow. However, the generalised method is applicable to any convenient approximation to the glottal tissue, whose equations of motion can be formally coupled to the volume flux Q via the general solution (13) of the aerodynamic sound problem. The successful extension of the method to more realistic vocal tract geometries and wall tissue properties will depend on a proper modelling of the low frequency acoustics of the system, an understanding of which is required to derive the corresponding compact Green's function.

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